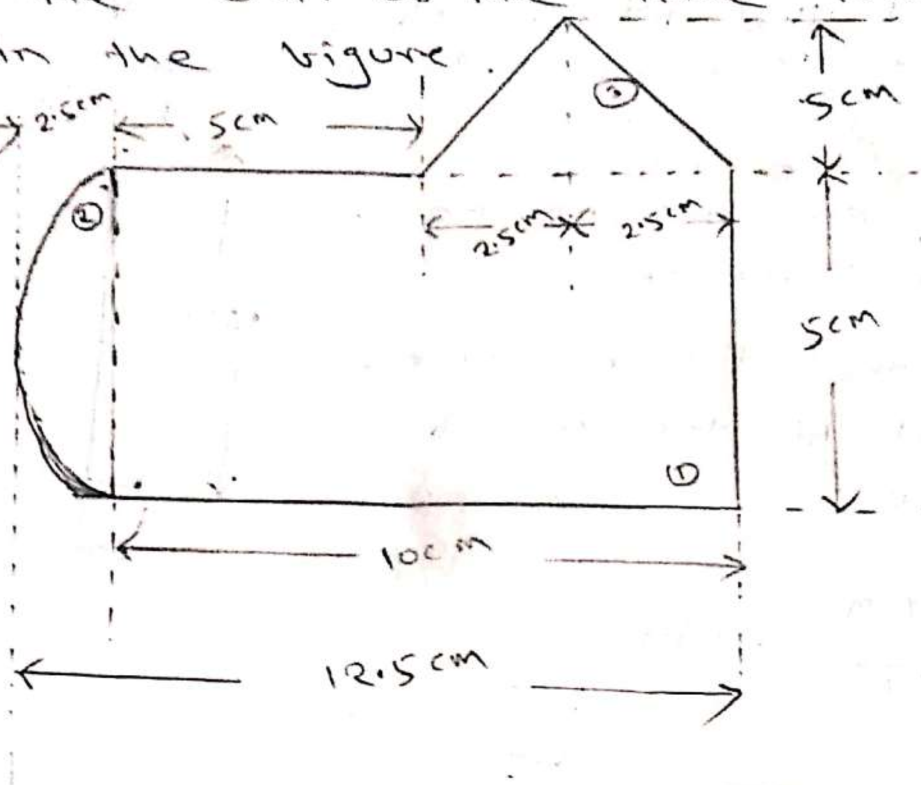
# CENTRE OF GRAVITY

(3) Determine the C.G. of the Plane lamina shown in the figure.




$a_1 = 10 \times 5$

$a_2 = \frac{\pi r^2}{2}$

$= \frac{\pi \times (2.5)^2}{2}$

$a_3 = \frac{1}{2} \times 5 \times 5$

$y_1 = 2.5$

$y_2 = 2.5$

$y_3 = 5 + \frac{5}{3}$

$x_1 = 2.5 + \frac{10}{2}$

$x_2 = r - \frac{4r}{3\pi}$

$= 2.5 - \frac{4 \times 2.5}{3\pi}$

$x_3 = 2.5 + 5 + 2.5$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = 3.22\text{cm}$

$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$

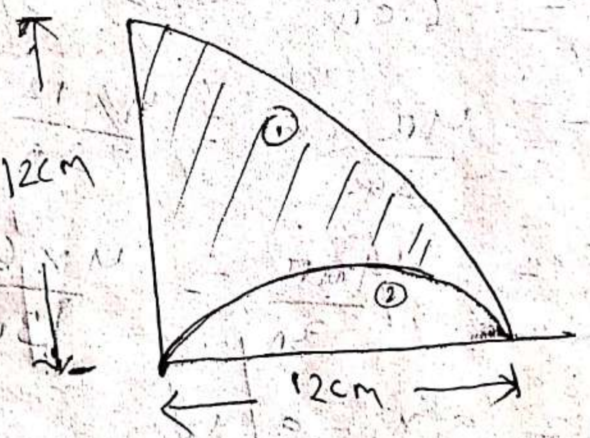
$= 7.11$ cm

(1) Centre of Gravity of sections with cut out holes

The C.G. of a section with a cut out hole is found out in the following ways

First find out the C.G. of the main section then deduct the cut out portion then we will get C.G. of the remaining portion.

e.g




Sol^n — let section (1) is completely a quarter circle from that a semi-circle i.e. section (2) will be removed by cutting from quarter circle

Semicircle dia = 12cm
Quarter circle radius = 12cm

$$a_1 = \frac{\pi r^2}{4}$$

$$= \frac{\pi \times (12)^2}{4}$$

$$= \pi \times \frac{144}{4}$$

$$= \pi \times 36 \text{ cm}^2$$

$$a_2 = \frac{\pi r^2}{2} = \frac{\pi \times (6)^2}{2}$$

$$= \frac{\pi \times 36}{2}$$

$$= 18\pi \text{ cm}^2$$

$$x_1 = \frac{4r}{3\pi} \text{ cm} = \frac{4 \times 12}{3\pi} = \frac{16}{\pi}$$

$$x_2 = 6 \text{ cm}$$

$$y_1 = \frac{4r}{3\pi} \text{ cm} = \frac{4 \times 12}{3\pi} = \frac{16}{\pi}$$

$$y_2 = \frac{4r}{3\pi} = \frac{4 \times 6}{3\pi} = \frac{8}{\pi}$$

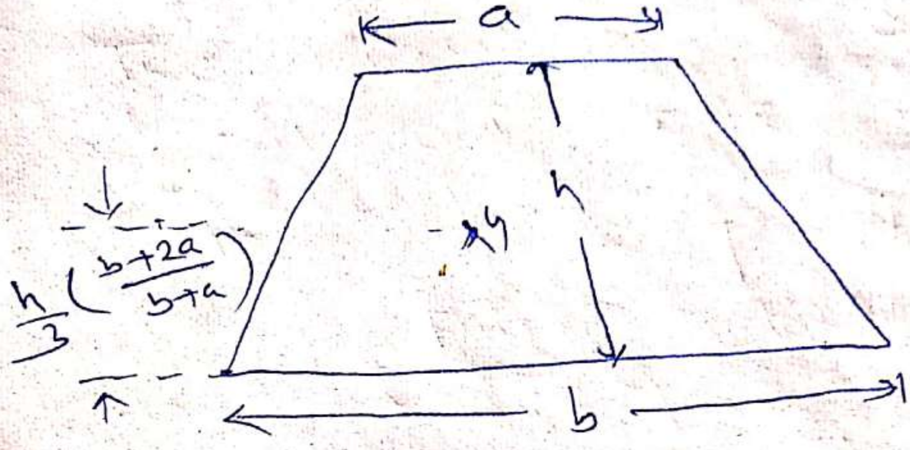
$$\therefore \bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$= \frac{\left(\pi \times 36 \times \frac{16}{\pi}\right) - 18\pi \times 6}{36\pi - 18\pi} = 4.17 \text{ cm}$$

$$\hat{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$$= \frac{\pi \times 36 \times \frac{16}{\pi} - \pi \times 18 \times \frac{8}{\pi}}{36\pi - 18\pi} = 7.64$$

Note: C.G. of a Trapezium




So here C.G. of a Trapizium with parallel sides 'a' and 'b' measured from the side 'b' (larger length)

is $\frac{h}{3}\left(\frac{b+2a}{b+a}\right)$

If C.G will measured from smaller side then that value will be

$$h - \frac{h}{3}\left(\frac{b+2a}{b+a}\right)$$

Area of trapezium $= h\left(\frac{b+a}{2}\right)$

15/04/11

# MOMENT OF INERTIA

We know moment of a force about a point is

O —x→ ↓P

force x perpendicular distance about that point

let $\boxed{\text{Moment} = P \times x}$

If again we take moment of that value about same point then we will get moment of the moment of a ~~force~~ a force or 2nd moment of force or it is called moment of inertia.

Similarly we can consider area or mass of a body

So 2nd moment of area about a point ~~is~~ or second moment of mass about a point ~~is~~ i.e called moment ~~of~~
all such 2nd moments are termed as ~~mass~~ moment of inertia.

## Moment of inertia of a Plane area :-

Assuming a plane area is composition of number of small areas say $a_1, a_2, a_3$ etc. at the distances $r_1, r_2, r_3$ etc. from a fixed line

then area moment of inertia or moment of inertia $= I = I_1 + I_2 + I_3 + \cdots$

$$= a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \cdots$$

$$= \sum a r^2$$

Fig-I

## Unit of M.I.

The unit of M.I. depends upon units of mass, area, length

(i) If mass is in kg distance is in m the M.I. is given $kg\text{-}m^2$

(ii) If area in $m^2$ length in m

the M.I. $= m^4$

## Radius of gyration :-

If the entire area 'A' of a given ~~axis~~ section be assumed to be concentrated at a certain point, at a distance 'K' from the given axis, such that-

$$AK^2 = a_1r_1^2 + a_2r_2^2 + a_3r_3^2 + \cdots$$

$$= \Sigma ar^2 \quad (\because \text{as shown in Fig-II})$$

$$\text{or } k^2 = \frac{\Sigma ar^2}{A}$$

$$K = \sqrt{\frac{\Sigma ar^2}{A}}$$

$$= \sqrt{I/A}$$

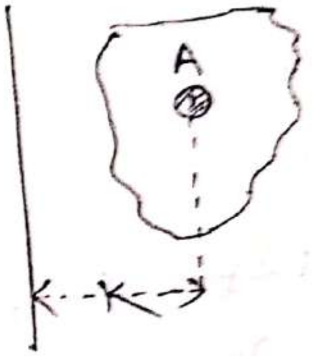
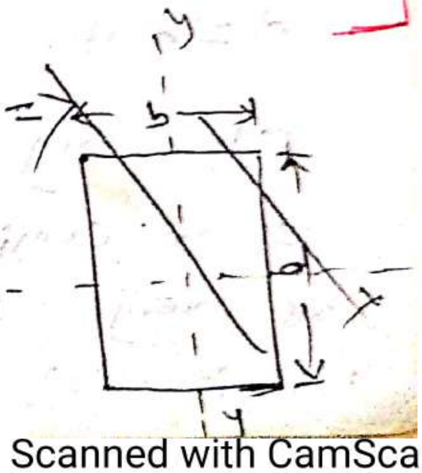



Fig-II

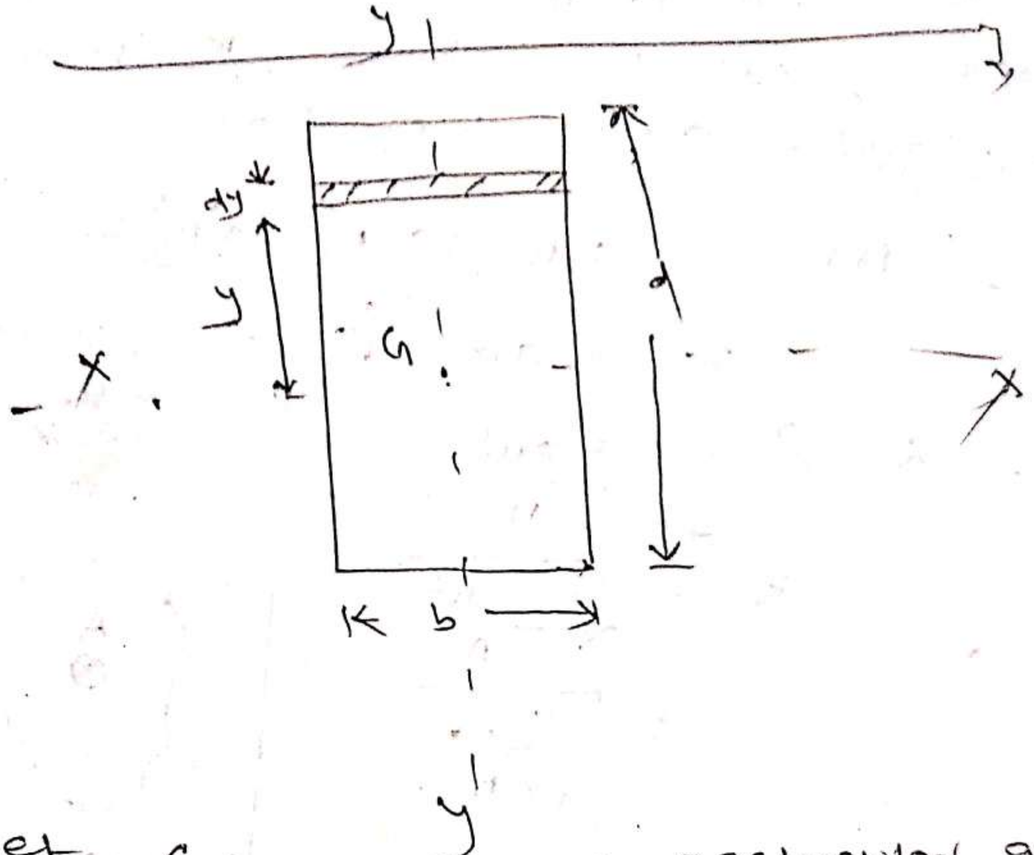
So this distance 'K' is called radius of gyration or radius of rotation.

[Thus radius of gyration is defined as the distance from an axis of reference where the whole mass (or area) of a body assumed to be cocentrated.]

Modulus of section :-



M.I. of a Rectangular area About centroidal axis - XX




Let G = C.G. of a rectangular area lying in the plane of the paper.

XX = an axis passing through G and parallel to width of the rectangle

YY = an axis passing through G and parallel to depth of the rectangle

b = width
d = depth

at any distance y from xx-axis let us consider an elementary strip of very small thickness dy which is parallel to xx-axis

The area of this elementary strip $= b \times dy = dA$

M.I. of the elementary area about $xx$ axis

$$= dA \times y^2 \qquad (\because dA = b \times dy)$$

$$= b \times dy \times y^2$$

$\therefore$ the required M.I. of the whole area about $xx$-axis is given by integrating it

$$I_{xx} = \int_{-d/2}^{+d/2} b \times dy \times y^2$$

$$= b \int_{-d/2}^{+d/2} y^2 . dy$$

$$= b \left. \frac{y^3}{3} \right]_{-d/2}^{d/2}$$

$$= \frac{b}{3} \left[ (d/2)^3 - (-d/2)^3 \right]$$

$$= b/3 \left( d^3/8 + d^3/8 \right)$$

$$= \frac{b}{3} \cdot 2d^3/8$$

$$\boxed{I_{xx} = bd^3/12}$$

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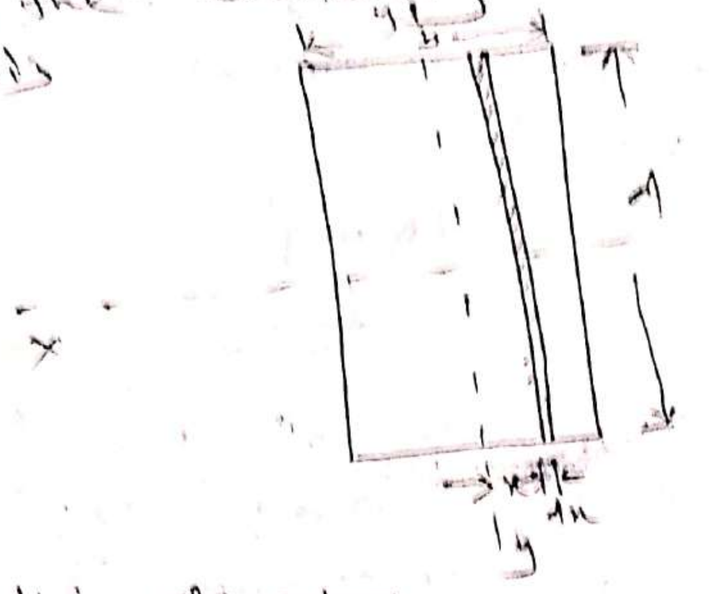
Similarly we can find out M.I. of the elementary strip about y-y axis




Area of the elementary strip = $\boxed{d \times dx = dA}$

M.I. of the elementary strip about yy axis

$$= dA \times x^2$$

$$= d \cdot dx \cdot x^2$$

∴ required M.I. of the whole area about yy-axis

$$I_{yy} = \int_{-b/2}^{+b/2} d \cdot dx \cdot x^2$$

$$= d \int_{-b/2}^{+b/2} dx \cdot x^2$$

$$= d \left. \frac{x^3}{3} \right]_{-b/2}^{+b/2}$$

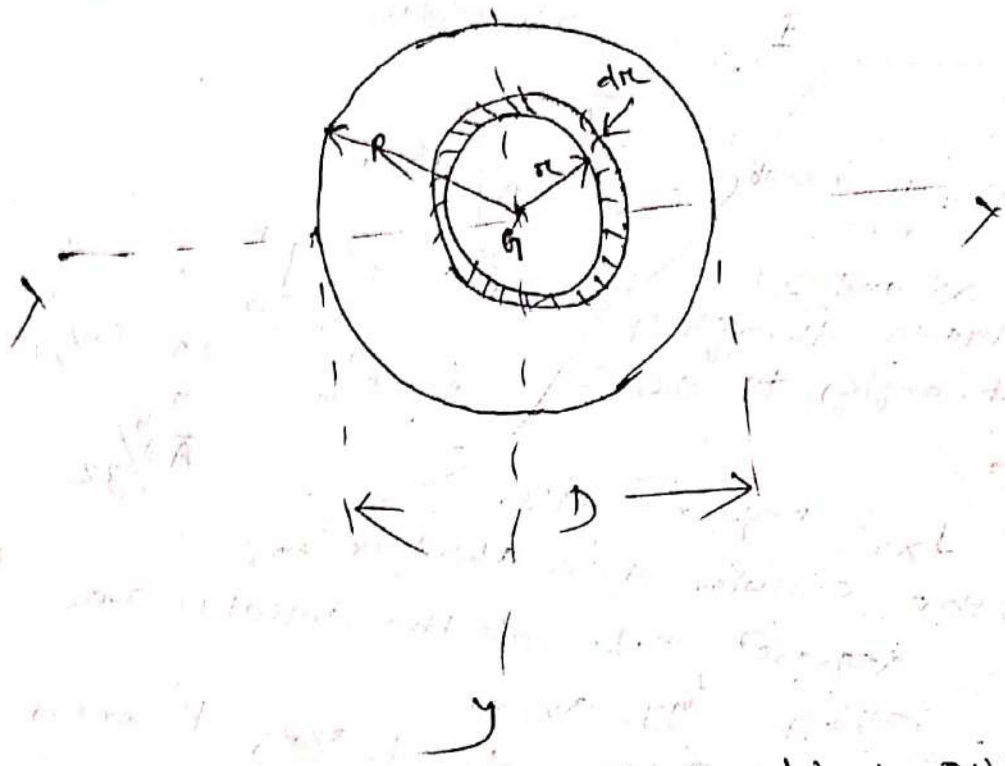
$$= \frac{d}{3}\left[ (b/2)^3 - (-b/2)^3 \right]$$

$$\frac{d}{3}\left[\frac{b^3}{8}+\frac{b^3}{8}\right]$$

$$= \frac{2db^3}{24}$$

$$\boxed{I_{yy} = \frac{db^3}{12}}$$

## M.I. of a Circular Area




First of all we have to find out the M.I. of the circular area about an axis ZZ which is not shown in fig & it is passing through G and ⊥ to the plane of the paper.

Let R = radius of circular area.

At any radius 'r' from the centre 'O' of the circle, let us

Consider an elementary annular strip of very small thickness $dr$

Then area of this elementary strip $= 2\pi r \times dr$

$\therefore$ M.I. of this elementary strip about zz axis $= 2\pi r . dr \times r^2$

$= 2\pi r^3 dr$

Then M.I. of whole circular Area about zz-axis is given by

$$I_{zz} = \int_0^R 2\pi r^3 dr$$

$$= 2\pi \int_0^R r^3 dr$$

$$= 2\pi \left[ \frac{r^4}{4} \right]_0^R$$

$$= 2\pi \times \frac{R^4}{4} = \frac{2\pi}{4} \left(\frac{D}{2}\right)^4$$

$$= \frac{\pi D^4}{32}$$

***

where

D = Dia of circular area.

Axes xx and yy are drawn through G at rt. angles to each other.

Let $I_{xx}$ = Required M.I. of the circular area about xx-axis **

$I_{yy}$ = Required M.I. of the circular area about yy-axis

We know according to ⊥ axes theorem

$$I_{zz} = I_{xx} + I_{yy}$$

but it is evident that for a circular section

$$I_{xx} = I_{yy}$$

$$\therefore I_{zz} = I_{xx} + I_{yy}$$

$$= I_{xx} + I_{xx}$$

$$= 2 I_{xx}$$

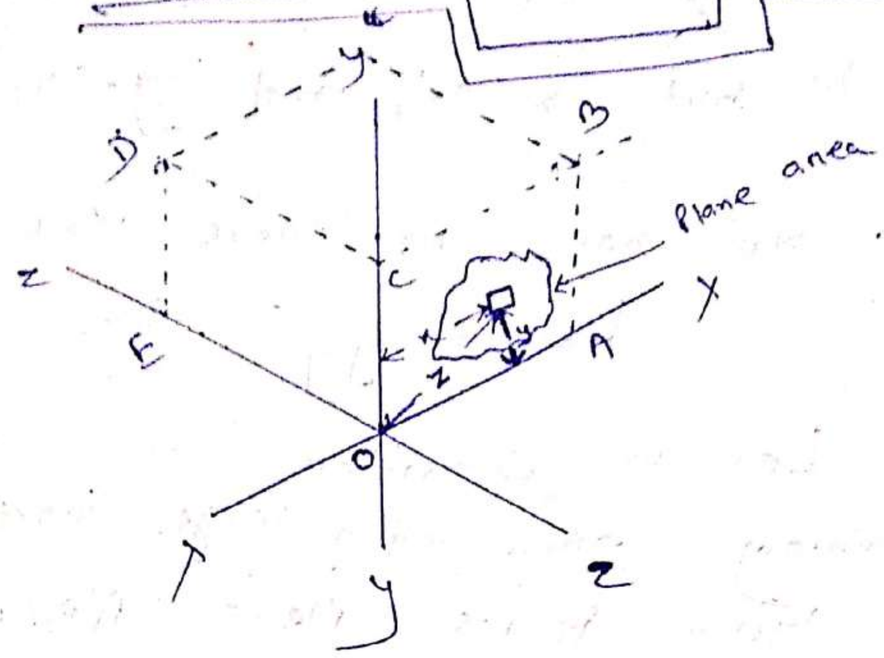
$$\text{or} \quad I_{xx} = \frac{I_{zz}}{2}$$

$$= \frac{\pi D^4 / 32}{2} = \frac{\pi D^4}{64}$$

Hence for circular area

$I_{xx} = I_{yy} = \pi D^4 / 64$

# THEOREM OF Perpendicular axes




Statement of Theorem :-

It states that the sum of the moments of inertia of an area about two mutually perpendicular and co-planar axes is equal to the moment of inertia of the area about an axis which is perpendicular to both the above axes.

i,e $I_{xx} + I_{yy} = I_{zz}$

Proof Let $I_{xx}$ = M.I. of an area about XX-axis

$I_{yy}$ = M.I. of an area about YY-axis which is at rt. angle to XX axis

$I_{zz}$ = M.I. of the area about zz axis which is at rt. angle to both xx axis and yy-axis.

So we have to prove that

$$I_{xx} + I_{yy} = I_{zz}$$

Let us consider an elementary area of a large plane area lying in the plane ABCO which is also contains xx axis and yy-axis

Let dA = magnitude of elementary area

x = distance of the elementary area ~~about~~ from yy-axis

y = distance of the elementary area from xx-axis.

Z = distance of the elementary area from zz-axis.

Now from the rt. angle triangle

$$z^2 = x^2 + y^2 \quad \text{———} (1)$$

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Multiplying both side of equation by $dA$ so we get

$$dA \cdot z^2 = dA\,x^2 + dA\,y^2$$

Now Integrating both side

$$\int dA\, z^2 = \int dA\, x^2 + \int dA\, y^2$$

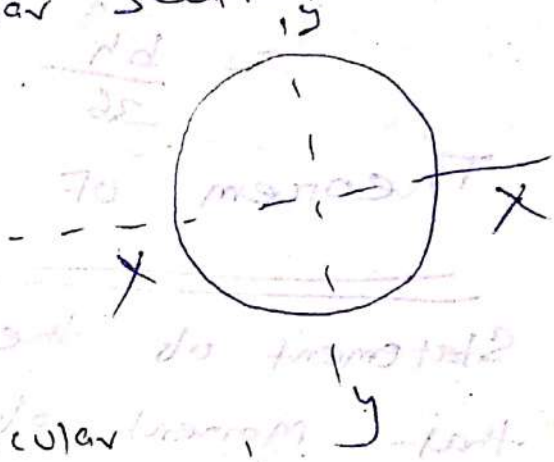
but we ~~know~~ have proved

$$I_{zz} = I_{yy} + I_{xx}$$

1) M.I. of circular section

$$I_{xx} = \frac{\pi d^4}{64}$$

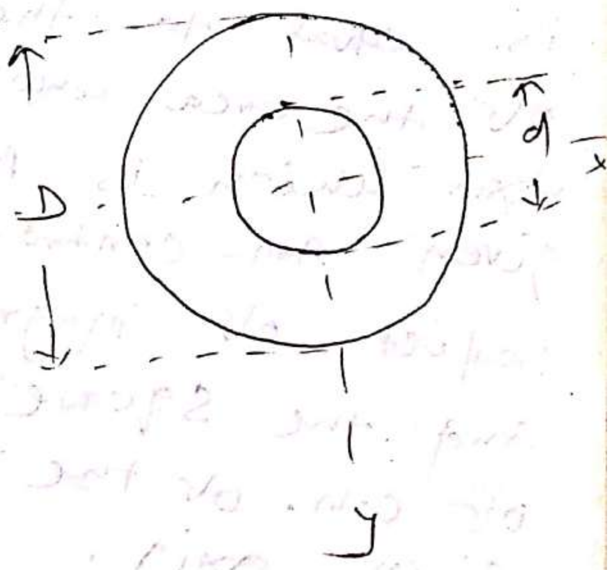
$$I_{yy} = \frac{\pi d^4}{64}$$





2) M.I of hollow circular section

$I_{xx}$ = M.I. of outer circle − M.I. of inner circle

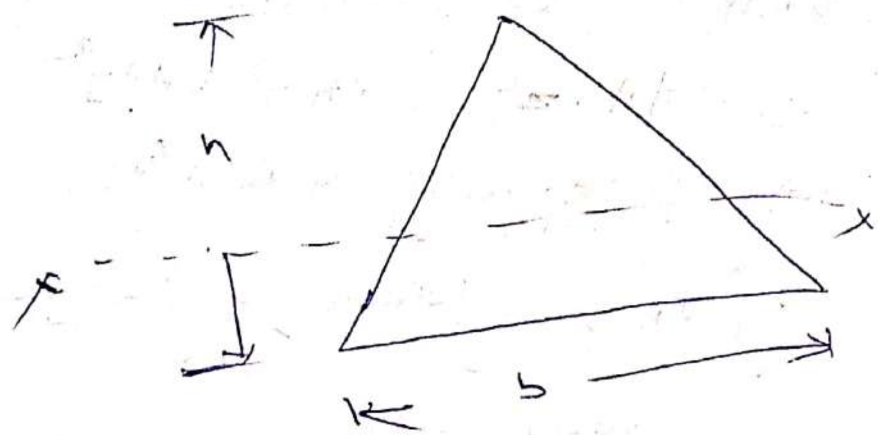




$$= \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

$$= \frac{\pi}{64}\left(D^4 - d^4\right)$$

3) M.I. of a triangular Area



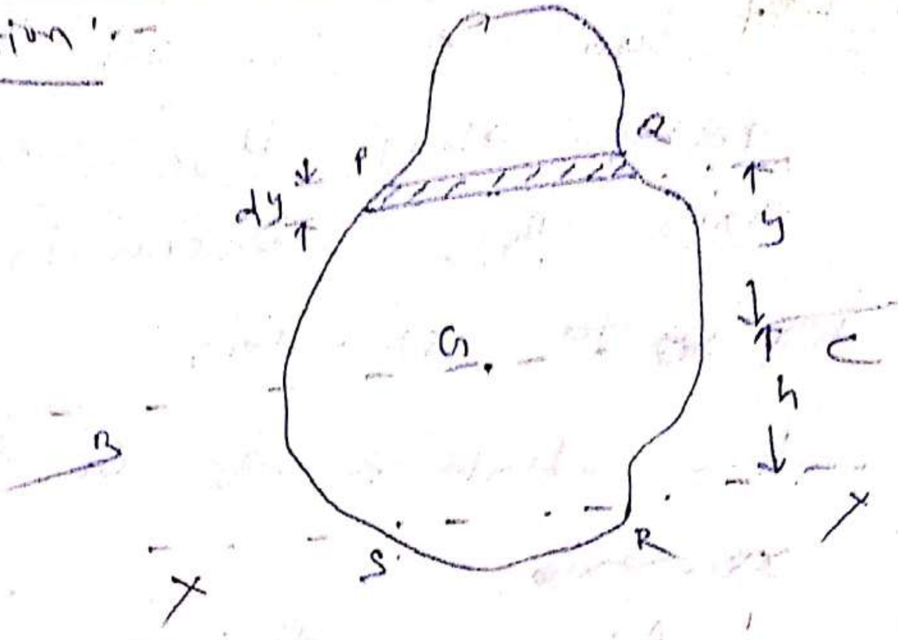

$\therefore$ $I_{xx}$ = required M.I. of the triangular area ABC about $xx$ axis

$$\therefore = \frac{bh^3}{36}$$

## Theorem of Parallel Axis :

Statement of theorem :- It states that moment of inertia of an area about a non-centroidal axis is equal to the moment of inertia of the area about its centroidal axis which is parallel to the given non-centroidal axis plus the product of magnitude of the area and the square of the distance of c.g. of the area from the given axis.

## Explanation :-




According to the above Statement-

$$I_{xx} = I_G + Ah^2$$

where $I_{xx}$ = Required M.I. ov an area about non-centroidal axis XX

$I_G = I_{BC}$ = M.I. ov the area about the axis BC which passes through the C.G. ov the area and parallel to XX axis

A = magnitude ov the whole area

h = distance ov C.G ov the area brom XX - axis.

Let 'G' be the C.G. ov an area PQRS and XY is parallel to BC and i,e called non-centroidal axis.

Let $I_{yy}$ = M.I. of the area PQRS about its centroidal axis BC which is parallel ~~to~~ to XX - axis

~~h = distance of G from XX - axis~~

At any distance y from BC axis let us consider an elementary strip of very small thickness dy

let dA = Area of elementary strip

So M.I. of the elementary strip about BC - axis

$= dA\, y^2$

M.I. of whole area PQRS about BC axis is given by $I_{BC} = \int dA\, y^2$ or i,e

$I_{G} = \int dA\, y^2$

Again M.I. of elementary Strip about XX - axis

$$= dA (y+h)^2$$

So M.I. of whole area PQRS about XX axis

$$= \int dA (y+h)^2$$

$$= \int dA (y^2+h^2+2yh)$$

$$= \int dA y^2 + \int h^2 dA + \int 2yh dA$$

$$= I_G + h^2 \int dA + 2h \int y dA$$

$$= I_G + Ah^2 + 2hA\bar{y}$$

$$(\because \int y dA = A\bar{y})$$

where $A$ = Area PQRS

$\bar{y}$ = distance of C.G of the area PQRS from BC axis. Since BC axis coincide with CG of the area PQRS So $\bar{y} = 0$

$$\therefore \boxed{I_{xx} = I_{GG} + Ah^2}$$

**Prob^m** — Find the M.I. of an Isosceles triangular section ABC with base 8 cm and height 6 cm about XX axis passing through the C.G of the section and the base AB.


**Sol^n**

1) M.I. about C.G of the section

We know

$$I_{xx} = \frac{bh^3}{36}$$

$$= \frac{8 \times (6)^3}{36}$$

$$= 48\ cm^4$$

2) M.I about AB

So using parallel axis theorem

$$I_{AB} = I_{xx} + Ah^2 = I_{GG} + Ah^2 \quad (\because I_{xx} = I_G)$$

$$= \frac{bh^3}{36} + Ah^2$$

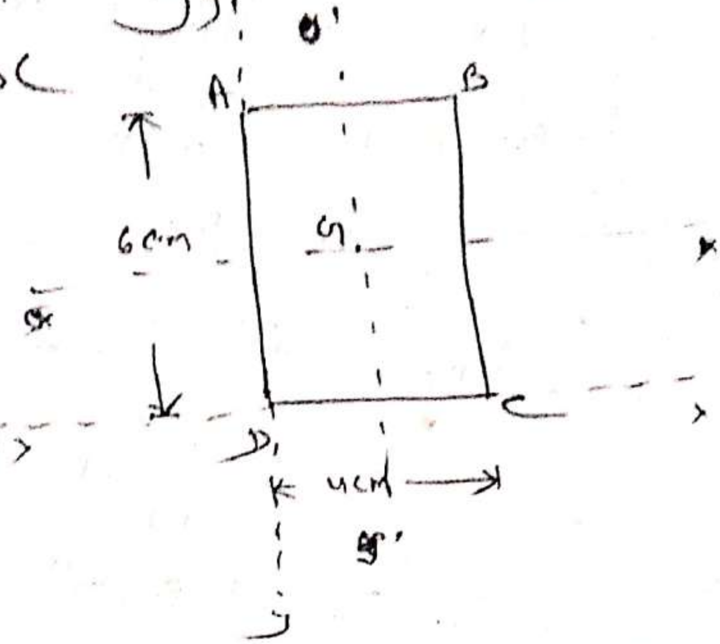
$$= 48 + \frac{1}{2} \times 8 \times (6) \times [\text{distance between } xx \text{ and } AB]^2$$

$$= 48 + \frac{1}{2} \times 8 \times 6 \times \left(\frac{6}{3}\right)^2$$

$$= 48 + \frac{1}{2} \times 8 \times 6 \times (2)^2$$

$$= 144\ cm^4 \quad (\text{Ans})$$

Prob^m - 2  Find the M.I. of rectangular Section shown in fig, about the face AB and BC




Sol^n

Given

$b = 4cm$
$d = 6cm$

we have to find out -> M.I. about AB and BC

So M.I. of the section about an axis through its c.g. and parallel to xx axis

$$I_{GGx} = \frac{bd^3}{12} = \frac{4 \times (6)^3}{12} = 72 cm^4$$

distance of c.g from AB = h = 3cm

① ∴ M.I. about the face AB

$$\Rightarrow I_{AB} = I_{Gx} + Ah^2$$
$$= \frac{bd^3}{12} + (4 \times 6) \times (3)^2$$
$$= 72 + 24 \times 9$$
$$= 288 cm^4$$

ⓘⓘ M.I. about the face BC

So first find out M.I. about c.g and parallel to yy axis

$$I_{GGy} = \frac{db^3}{12} = \frac{6 \times 4^3}{12} = 32 cm^4$$

by using parallel axis theorem

$$I_{BC} = I_{yy} + Ah^2$$

where $h$ = distance between c.g and base BC = 2 c.m

$$\therefore I_{BC} = \frac{db^3}{12} + Ah^2$$

$$= \frac{6 \times (4)^3}{12} + 24 \times (2)^2$$

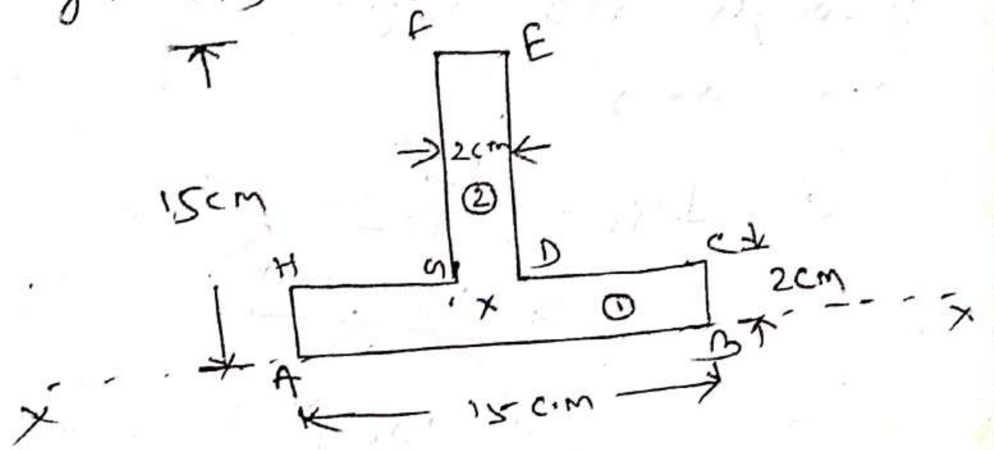
$$= 128 \, cm^4$$

## MOMENT OF INERTIA OF COMPOSITE SECTION

The M.I. of composite sections may be found out by dividing the whole section in to different section which is convenient to us and then calculating M.I. about their respective own c.g. Then that M.I. transfer to the required axis or the section by using parallel axis theorem i.e

$$I_{AB} = I_{xx} + Ah^2$$

Prob^m - 1 A T-section is 15×15×2 cm as shown in fig. Calculate the M.I. of the section about XX-axis parallel to the base of Tee passing through its C.G.




Sol^n First of all we have to find out C.G. of the section. Since the section is symmetrical about YY-axis so find out $\bar{y}$ which is distance between face AB and c.g. of the section.

Split up the section in to two parts

① Area ABCH

$a_1 = 15 \times 2 = 30\,cm^2$

$y_1 = 1\,cm$

② Area DEFG

$a_2 = 13 \times 2 = 26\,cm^2$

$y_2 = 2 + 6.5 = 8.5\,cm$

Using the relation

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= 4.48 \text{ cm}$$

(∵ C.G is at a distance of 4.48 from base AB)

Now M.I. of rectangle ① through its C.G. and parallel to XX-axis

$$\therefore I_{G_1} = \frac{bd^3}{12}$$

$$= \frac{15 \times (2)^3}{12}$$

$$= 10 \text{ cm}^4$$

So distance of C.G. of rectangle ① from X-X axis $= h_1$ (say)

$$= 4.48 - 1$$

$$h_1 = 3.48 \text{ cm}$$

∴ M.I. of rectangle 1 about X-X axis

$$I_1 = I_{G_1} + A_1 h_1^2$$

$$= 10 + (15 \times 2) \times (3.48)^2$$

$$= 373.3 \text{ cm}^4$$

Similarly M.I. of rectangle ② through its C.g. and parallel to XX-axis

$$I_{cg_2} = \frac{bd^3}{12}$$

$$= \frac{2 \times (13)^3}{12}$$

$$= 366.17 \text{ cm}^4$$

now distance of C.G of rectangle ② from xy axis $= h_2$ (say)

$$h_2 = 8.5 - 4.48$$

$$= 4.02 \text{ cm}$$

∴ M.I. of rectangle ② about xy axis :

$$I_2 = I_{cg_2} + A_2 h_2^2$$

$$= 366.17 + (2 \times 13) \times (4.02)^2$$

$$= 786.33 \text{ cm}^4$$

∴ M.I. of the whole section about xy - axis

$$I_{xx} = I_1 + I_2$$

$$= 373.3 + 786.33$$

$$= 1159.63 \text{ cm}^4$$

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Procedure to finding the M.I. of composite Section

① Split of the section in to two or three rectangles (e.g T-section or I-section) respectively.

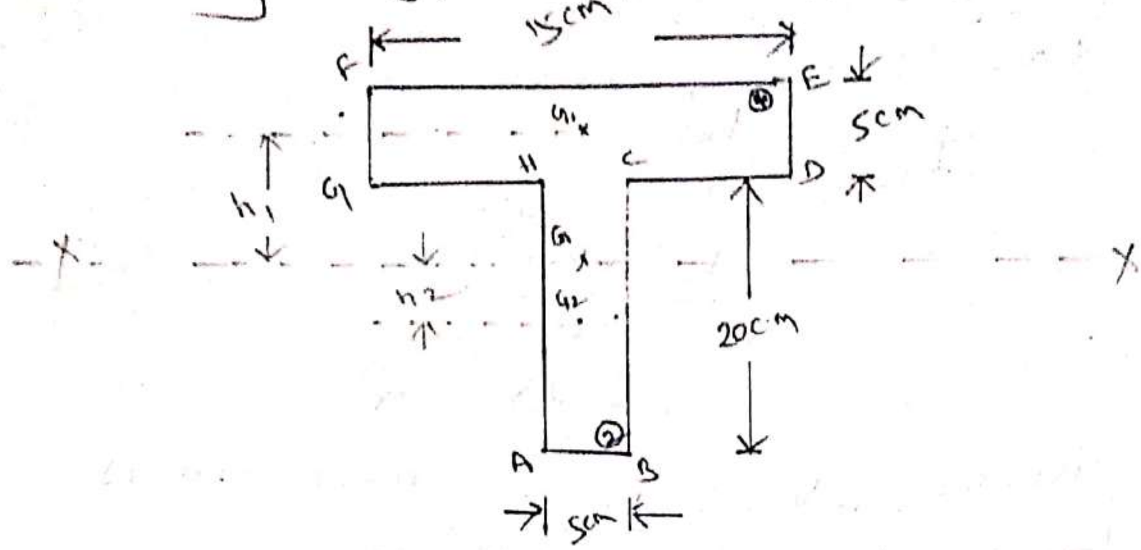
② find out c.g. of the section taking bottom face or extreme left side as reference axis,

③ find out M.I. of different rectangles about its own c.g and parallel to XX axis or ~~yy axis~~ yy-axis, according to question given.

④ Transfer the M.I. of each section or each rectangle to c.g. of the main ~~composite~~ section by using parallel axis theorem.

⑤ Summing up all the M.I. of different rectangles then we will get M.I. of main Section i,e $I_{XX} = I_1 + I_2 + I_3 + \ldots$

(3) 2008 Determine M.I. ($I_{xx}$) of the following fig.




The section is symetric about yy-axis so find out $\bar{y}$. Let $\bar{y}$ be the distance between c.g. of the section and face AB

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$\therefore$ $a_1 = 15 \times 5 = 75 \text{ cm}^2$

$a_2 = 20 \times 5 = 100 \text{ cm}^2$

$y_1 = 20 + 2.5$

$= 22.5 \text{ cm}$

$y_2 = 10 \text{cm}$

$$\therefore \bar{y} = \frac{(75 \times 22.5) + (100 \times 10)}{75 + 100}$$

$$= \frac{1687.5 + 1000}{175}$$

$= 15.35$ cm

Now M.I. of the rectangle (1) through its c.g. and parallel to XX - axis

$$I_{G_1} = \frac{bd^3}{12}$$

$$= \frac{15 \times (5)^3}{12}$$

$$= 156.25 \text{ cm}^4$$

Distance of c.g. of rectangle (1) from XX - axis = say

$$h_1 = 22.5 - 15.35$$

$$\boxed{h_1 = 7.15 \text{ cm}}$$

M.I. of rectangle (1) about XX - axis

$$I_1 = I_{G_1} + A_1 h_1^2$$

$$= 156.25 + (15 \times 5) \times (7.15)^2$$

$$= 3990.43 \text{ cm}^4$$

Similarly M.I. of rectangle (2) through its c.g. and parallel to XX - axis

$$I_{G_2} = \frac{bd^3}{12}$$

$$= \frac{5 \times (20)^3}{12}$$

$$= 3333.33 \text{ cm}^4$$

Distance of c.g. of rectangle (2) from XX - axis = $h_2$ (say)

$h_2 = 15.35 - 10$

$= 5.35$ cm

∴ M.I. of rectangle (2) about xx axis

$I_2 = I_{G_2} + A_2 h_2^2$

$= 3333.33 + (20\times5)(5.35)^2$

$= 6195.58\ cm^4$

∴ M.I. of the whole section about xx- axis

$$\boxed{I_{xx} = I_1 + I_2}$$

$= 3990.43 + 6195.58$

$$\boxed{I_{xx} = 10186.01\ cm^4}$$

m ... of channel

18/03/2020

# SIMPLE LIFTING MACHINE

In ancient time thousands of slaves had to be arranged to carry a load. But now a days due to advancement of technology it is very easy to handle a heavy load by a single ~~man~~ person.

Simple machine :- It is a such type of device by which some useful work can be obtained by applying some effort.

Compound machine :- It is the arrangement of number of simple machine by which some useful work can be obtained by applying less effort than simple machine.

Lifting machine :-> It is a mechanical device which is used to lift a a heavy load 'W' by applying very less effort or force.

Mechanical advantage :- (M.A) It is defined as ratio of weight or load lifted ~~by~~ applying to the effort i.e $M.A = \frac{W}{P}$

where W = weight lifted
P = effort applied

It is having no unit.

## Input of a Machine :-

Input is workdone on the Machine i.e. we know WD = F × S

So input = Effort applied × distance through which effort moved

$= P \times y$

where P = effort

y = distance moved by effort

## Output of Machine :-

It is workdone by the Machine i.e. WD = load lifted × distance moved by that load

$= W \times x$

where W = weight

x = distance moved by weight or load

Efficiency => It is the ratio of output to the input

So Efficiency = $\eta = \frac{\text{output}}{\text{Input}} \times 100$ %

Ideal machine :=>

It is such type of machine in which efficiency is 100%.

i,e ~~[illegible]~~ Input = output

So $\eta = \frac{\text{output}}{\text{Input}} \times 100$ %

$= 100\%$.

(It is impossible for practical point of view)

Velocity ratio :=>

It is the ratio of distance moved by effort to the distance moved by load

So $V.R = \frac{y}{x}$

Relation ship between ~~...~~ $\eta$, M.A and V.R of a lifting M/C :-

let W = load to be lifted

P = effort

y = distance moved by effort

x = distance moved by load

$\therefore$ $V \cdot R = \frac{y}{x}$

$M \cdot A = \frac{W}{P}$

So we know

In put of a Machine

$= P \times y$

Out put of a machine

$= W \times x$

$\therefore \eta = \frac{\text{Out put}}{\text{In put}}$

$= \frac{W \times x}{P \times y}$

$= \frac{W/P}{y/x}$

$$\boxed{\eta = \frac{M A}{V \cdot R}}$$

Prob^m — In a certain weight lifting machine a weight of 1000 kg is lifted by an effort of 25 kg. While the weight moves up by 10 c.m the point of application of effort moves by 800 c.m. Find (i) M.A (ii) V.R (iii) efficiency of the M/c

Sol^n

$$M.A = W/P = \frac{1000}{25} = 40$$

$$V.R = y/x = \frac{800}{10} = 80$$

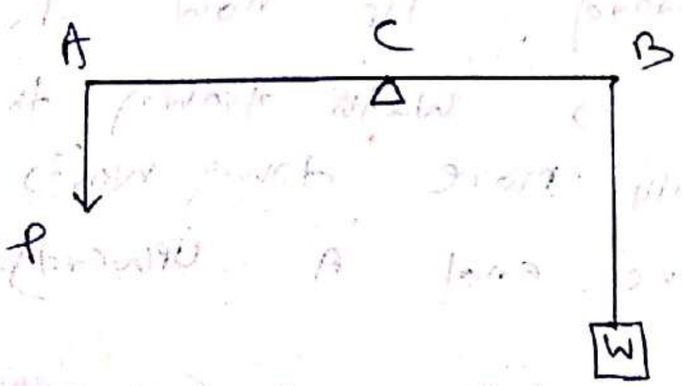
$$\eta = \frac{M.A}{V.R}$$

$$= \frac{40}{80} = \frac{1}{2}$$

$$= 0.5 \times 100$$

$$= 50\%$$

Reversibility of a machine :-



With reference to figure shown above a lever arrangement which is a kind of lifting m/c : where effort P is applied at end 'A' to lift the load 'W' at end 'B'

AB is called a lever

'C' is called fulcrum

AC is called effort Arm

BC is called load Arm

Generally effort arm > load Arm

So here when the effort 'P' is applied at the end 'A' to lift the load 'W' the end 'A' tends to move down wards and W tends to rise up ward. If now the effort 'P' is with drawn the end 'B' will move down wards raising the end 'A' upwards

And we say that the machine AB is reversible

If However the load W remains static even after withdrawal of effort P the machine is called Non-reversible or Self locking.

So a reversible M/c is that M/c in which the load moves in the reverse direction after withdrawal of the effort.

Condition for the reversibility of a machine

Consider a reversible machine in which

Let W = load lifted by the M/c

P = effort required to lift the load

y = Distance moved by the effort

x = Distance moved by the load

So input of the machine

$= P \times y$ — (i)

out put of the M/c $= W \times x$ — (ii)

We know that ~~Axe~~ ~~to~~ M/c friction

= Input − output

$= P \times y - W \times x$ — (iii)

K/Little ~~No~~

For a reversible M/c the out put of the M/c should be more than the

Machine function i.e when the effort 'p' is zero

So $W \times x > P \times y - W \times x$

$\Rightarrow (W \times x) + (W \times x) > P \times y$

$\Rightarrow 2(W \times x) > P \times y$

$$\Rightarrow \frac{W \times x}{P \times y} > \frac{1}{2}$$

$$\Rightarrow \frac{W/P}{y/x} > \frac{1}{2}$$

$$\frac{MA}{V.R} > \frac{1}{2}$$

$$\eta > \frac{1}{2}$$

$\Rightarrow \eta > 0.5$ or $50\%$.

Hence here we conclude that for a reversible machine $\eta$ is more than $50\%$.

## Self locking M/C :-

Some times the M/C is not capable of doing any work in reverse direction i.e after removal of effort, such a

M/c is called non- reversible or self locking M/c, So at that time 'η' should not be more than 50%.

## LAW OF LIFTING M/c

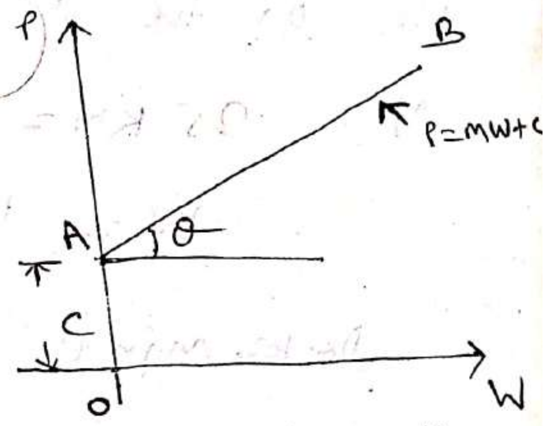
A lifting M/c follows a general law given by the equation

$$P = mW + c$$ where

P = effort applied

W = load lifted

m = a constant i.e which is equal to slope of the line AB




C = another constant.

This is the equation of a straight line

$y = mx + c$

Here m is the gradient of the St. line i.e $m = \tan\theta$

where θ = angle made by the St. line with x-axis which represents W. c is the intercept on the y-axis which represents 'p'

'c' represents the effort required to move the machine and over come friction at initial condition.

Probm

$P = 15 KN$

$W = 720 KN$

what is the $M.A = ?$ ?

if the $\eta = 60\% = 0.6$

find $V.R = ?$

if on the same m/c an effort of $25 KN = P$

$W = 1320 KN$

Determine $\eta = ?$

2nd case

$$M.A = \frac{load}{effort}$$

$$= \frac{1320}{25} = 52.8$$

∴ Regarding V.R. it will be constant for a particular m/c

so $\eta = \frac{MA}{V.R}$

$\therefore \eta = \dfrac{52.8}{V.R}$

Law of machine

$P = mW + C$

taking First condition

$15 = m \times 720 + C$ —— (i)

taking 2nd condition

$25 = m \times 1320 + C$ —— (ii)

Now solving eq (i) & (ii) we get

$m = 1/60$

$C = 3 \text{ KN}$

$\therefore$ Required Law of machine is

$P = mW + C$

$\boxed{P = \frac{1}{60} W + 3}$ Ans

## Some FORMULAE

1) Max^m^ mechanical advantage of a lifting M/C

we know $M.A = \dfrac{W}{P}$

for max^m^ M.A. substituting the value of $P = mW + C$ in the above equation we get

$$M.A = \frac{W}{P}$$

$$= \frac{W}{mW + C}$$

$$= \frac{W/W}{\frac{mW}{W} + \frac{C}{W}}$$ (dividing both Neumerator & denominator with 'W' weget)

$$= \frac{1}{m + \frac{C}{W}}$$

$$\boxed{\text{Max}^m\ M.A = \frac{1}{m}}$$ (neglecting $\frac{C}{W}$ because it is a small quantity)

2) Max$^m$. efficiency of a lifting m/c

we know $\eta = \frac{M.A}{V.R}$

$$= \frac{W/P}{V.R}$$

$$= \frac{W}{P(V/R)}$$

for max$^m$. efficiency putting the value of max$^m$. M.A. we get

$$\text{max}^m.\ \eta = \frac{1/m}{V.R}$$

$$\boxed{\text{Max}^m.\ \eta = \frac{1}{m \times V.R}}$$

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